

Three-gluon contribution to the single spin asymmetry in Drell-Yan and direct-photon processes

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Abstract

We derive the single-spin-dependent cross section for the Drell-Yan lepton-pair production and the direct-photon production in the pp -collision induced by the twist-3 three-gluon correlation functions in the transversely polarized nucleon in the leading order with respect to the QCD coupling constant. Combined with the contribution from the twist-3 quark-gluon correlation functions in the literature, this completes the twist-3 cross section for these processes. We also present a model calculation of the asymmetry for the direct photon production at the RHIC energy, demonstrating the sensitivity of the asymmetry to the form of the three-gluon correlation functions. In particular, we show that the asymmetry in the backward direction of the polarized nucleon is determined by the small- x behavior of the correlation functions.

1 Introduction

The single transverse-spin asymmetry (SSA) in the Drell-Yan lepton-pair production and the direct-photon production provides us with an ideal tool to investigate the multi-parton correlations and the orbital motion in the nucleon. (See [1] for a review.) The SSAs in these processes are caused solely by those effects in the initial nucleons, while the SSAs in hadron productions in pp collisions and semi-inclusive deep-inelastic scattering (SIDIS) receive potential contribution also from the multiparton correlations in the fragmentation process. For the production of the (virtual) photons with large transverse momentum, the process can be analyzed in the framework of the collinear factorization and the SSA appears as a leading twist-3 observable as a consequence of the quark-gluon correlations [2]-[18] and the multi-gluon correlations [19]-[26] in the transversely polarized nucleon. By now there have been several studies on the effect of the quark-gluon correlations on SSAs in these processes [3, 8, 12, 17, 18], and the corresponding twist-3 cross section formula in the leading-order QCD has been completed [17].

As for the multi-gluon correlations, the formalism for calculating the single-spin-dependent cross section induced by the three-gluon correlation functions has recently been established in [22, 23, 24], and its contributions to the D -meson production in SIDIS and the pp collision have been derived.¹ Since the gluon-photon and gluon-gluon fusion processes are the driving processes for the heavy meson productions, the corresponding SSAs are particularly useful to probe the three-gluon correlations. Experimentally, the measurement of the SSA for D - and J/ψ - production is ongoing at RHIC in BNL [27, 28], from which we hope to extract the form of the three-gluon correlation functions. With the use of the extracted three-gluon correlation functions for the SSAs in the Drell-Yan and the direct photon production in the pp collision, one can study their impact on the SSA in these processes. If it turns out that the SSAs in these processes are sensitive to the form of the three-gluon correlations, combined analysis of SSAs in D -meson productions and these processes will give more information on the form of both quark-gluon and multigluon correlation functions.

The purpose of this paper is to derive the twist-3 single-spin-dependent cross section for the Drell-Yan and the direct-photon production processes induced by the three-gluon correlation functions in the transversely polarized nucleon, applying the formalism developed in [22].² We shall also present a numerical calculation of the asymmetry A_N^γ for the direct photon production using the models for the three-gluon correlation functions extracted from the RHIC preliminary data of $p^\uparrow p \rightarrow DX$. We will see that the three-gluon contribution to A_N^γ is negligible in the forward region of the polarized proton but can be substantial in the backward region, in particular, the A_N^γ in the backward region is sensitive to the relative sign of the two three-gluon functions and the small- x behavior of the functions.

The rest of this paper is organized as follows: In Sec. 2, we summarize the complete set of the three-gluon correlation functions in the transversely polarized nucleon used in the

¹There had been earlier studies on the contribution of the three-gluon correlation functions to SIDIS [20] and $pp \rightarrow DX$ [21]. However, the formalism used there differs from [22] and the result also differs from those in [22, 24].

²This paper presents the full detail of the result presented in the conference proceedings [25, 26].

present analysis. In Sec. 3, we derive the twist-3 single-spin-dependent cross section for the Drel-Yan process. For the derivation, we apply the master formula developed in [23]. In Sec. 4, we present the twist-3 cross section for the direct-photon process by taking the real-photon limit of the result obtained in Sec. 3. We also present a model calculation of the asymmetry for this process $A_N^\gamma \equiv \Delta\sigma^\gamma/\sigma^\gamma$ at the RHIC energy. Sec. 5 is devoted to a brief summary.

2 Three-gluon correlation functions in the transversely polarized nucleon

As clarified in [29, 30, 22], there are two independent three-gluon correlation functions in the transversely polarized nucleon, $O(x_1, x_2)$ and $N(x_1, x_2)$, which are the Lorentz-scalar functions of the longitudinal momentum fractions x_1 and x_2 , defined as

$$\begin{aligned} O^{\alpha\beta\gamma}(x_1, x_2) &= -g(i)^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS | d_{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) | pS \rangle \\ &= 2iM_N \left[O(x_1, x_2) g^{\alpha\beta} \epsilon^{\gamma p n S_\perp} + O(x_2, x_2 - x_1) g^{\beta\gamma} \epsilon^{\alpha p n S_\perp} + O(x_1, x_1 - x_2) g^{\gamma\alpha} \epsilon^{\beta p n S_\perp} \right], \quad (1) \end{aligned}$$

$$\begin{aligned} N^{\alpha\beta\gamma}(x_1, x_2) &= -g(i)^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS | i f_{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) | pS \rangle \\ &= 2iM_N \left[N(x_1, x_2) g^{\alpha\beta} \epsilon^{\gamma p n S_\perp} - N(x_2, x_2 - x_1) g^{\beta\gamma} \epsilon^{\alpha p n S_\perp} - N(x_1, x_1 - x_2) g^{\gamma\alpha} \epsilon^{\beta p n S_\perp} \right], \quad (2) \end{aligned}$$

where $F_a^{\alpha\beta} \equiv \partial^\alpha A_a^\beta - \partial^\beta A_a^\alpha + g f_{abc} A_b^\alpha A_c^\beta$ is the gluon's field strength, and we used the notation $F_a^{\alpha n} \equiv F_a^{\alpha\beta} n_\beta$ and $\epsilon^{\alpha p n S_\perp} \equiv \epsilon^{\alpha\mu\nu\lambda} p_\mu n_\nu S_{\perp\lambda}$ with the convention $\epsilon_{0123} = 1$. d^{bca} and f^{bca} are the symmetric and anti-symmetric structure constants of the color SU(3) group, and we have suppressed the gauge-link operators which ensure the gauge invariance. p is the nucleon momentum, and S_\perp is the transverse spin vector of the nucleon normalized as $S_\perp^2 = -1$. In the twist-3 accuracy, p can be regarded as lightlike ($p^2 = 0$), and n is another lightlike vector satisfying $p \cdot n = 1$. To be specific, we set $p^\mu = (p^+, 0, \mathbf{0}_\perp)$, $n^\mu = (0, n^-, \mathbf{0}_\perp)$, and $S_\perp^\mu = (0, 0, \mathbf{S}_\perp)$. The nucleon mass M_N is introduced to define $O(x_1, x_2)$ and $N(x_1, x_2)$ dimensionless. The decomposition (1) and (2) takes into account all the constraints from hermiticity, invariance under the parity- and time-reversal transformations and the permutation symmetry among the participating three gluon-fields. The functions $O(x_1, x_2)$ and $N(x_1, x_2)$ are real and have the following symmetry properties,

$$O(x_1, x_2) = O(x_2, x_1), \quad O(x_1, x_2) = O(-x_1, -x_2), \quad (3)$$

$$N(x_1, x_2) = N(x_2, x_1), \quad N(x_1, x_2) = -N(-x_1, -x_2). \quad (4)$$

3 Drell-Yan Process

3.1 Twist-2 unpolarized cross section

Before discussing the single-spin-dependent cross section, we present here the twist-2 unpolarized cross section for the Drell-Yan process, $p(p) + p(p') \rightarrow \gamma^*(q) + X$, which constitutes the denominator of the asymmetry $A_N^{DY} \equiv \Delta\sigma^{DY}/\sigma^{DY}$. The corresponding partonic hard cross section in the leading order QCD (LO) is obtained from the Feynman diagrams shown in Fig. 1. The phase space factor for the virtual photon d^4q is given in terms of the rapidity $y = \frac{1}{2} \ln(q^+/q^-)$, the squared-mass of the lepton-pair $Q^2 = q^2$ and the transverse momentum \vec{q}_\perp as $d^4q = \frac{1}{2} dQ^2 dy d^2\vec{q}_\perp$. The unpolarized cross section is obtained by taking the trace of the hadronic tensor $W^{\mu\nu}(p, p', q)$ as

$$\begin{aligned} \frac{d\sigma^{DY}}{dQ^2 dy d^2\vec{q}_\perp} &= \frac{\alpha_{em}^2}{3\pi S Q^2} (-W^\mu_\mu(p, p', q)) \\ &= \frac{\alpha_{em}^2 \alpha_s}{3\pi S Q^2} \int \frac{dx}{x} \int \frac{dx'}{x'} \sum_q e_q^2 [f_q(x) f_{\bar{q}}(x') \hat{\sigma}_{q\bar{q}} + G(x) f_q(x') \hat{\sigma}_{gq} \\ &\quad + f_q(x) G(x') \hat{\sigma}_{qg}] \delta(\hat{s} + \hat{t} + \hat{u} - Q^2), \end{aligned} \quad (5)$$

where S is the squared center-of-mass energy, $\alpha_{em} \simeq 1/137$ is the QED coupling constant, $\alpha_s = g^2/(4\pi)$ is the strong coupling constant, $e_u = 2/3$ and $e_d = -1/3$ etc are the electric charge of each quark flavor, and $f_q(x)$ and $G(x)$ are, respectively, the unpolarized quark distribution with flavor q and the gluon distribution. \sum_q denotes the sum over all quark and antiquark flavors. In (5), \hat{s} , \hat{t} and \hat{u} are the Mandelstam variables in the parton level defined as

$$\hat{s} = (xp + x'p')^2, \quad \hat{t} = (xp - q)^2, \quad \hat{u} = (x'p' - q)^2, \quad (6)$$

and the LO partonic hard cross sections are given by

$$\begin{cases} \hat{\sigma}_{q\bar{q}} = \frac{2C_F}{N} \left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2Q^2\hat{s}}{\hat{t}\hat{u}} \right), \\ \hat{\sigma}_{gq} = -\frac{1}{N} \left(\frac{\hat{s}}{\hat{u}} + \frac{\hat{u}}{\hat{s}} + \frac{2Q^2\hat{t}}{\hat{s}\hat{u}} \right), \\ \hat{\sigma}_{qg} = \hat{\sigma}_{gq}(\hat{t} \leftrightarrow \hat{u}), \end{cases} \quad (7)$$

where $N = 3$ denotes the number of colors for quarks. Here $\hat{\sigma}_{q\bar{q}}$ and $\hat{\sigma}_{gq}$ are, respectively, obtained from Fig. 1(a) and (b). In the next section, we will use the relation between the twist-3 cross section and the $gq \rightarrow \gamma^*q$ hard scattering cross section. For this purpose, we introduce the partonic hard part $\mathcal{H}_{gq, \alpha\beta}^{ab}$ for the gluon-quark scattering channel by the relation

$$\left(-\frac{1}{2} g_\perp^{\alpha\beta} \right) \frac{1}{(N^2 - 1)} \delta_{ab} \mathcal{H}_{gq, \alpha\beta}^{ab}(xp, x'p', q) = \hat{\sigma}_{gq}(\hat{s}, \hat{t}, \hat{u}, Q^2) \delta(\hat{s} + \hat{t} + \hat{u} - Q^2), \quad (8)$$

where the factors $-1/2g_{\perp}^{\alpha\beta}$ with $g_{\perp}^{\alpha\beta} = g^{\alpha\beta} - p^{\alpha}n^{\beta} - p^{\beta}n^{\alpha}$ and $1/(N^2 - 1)\delta_{ab}$ are, respectively, associated with the Lorenz and color projections for the unpolarized gluon density to obtain the cross section in this channel.

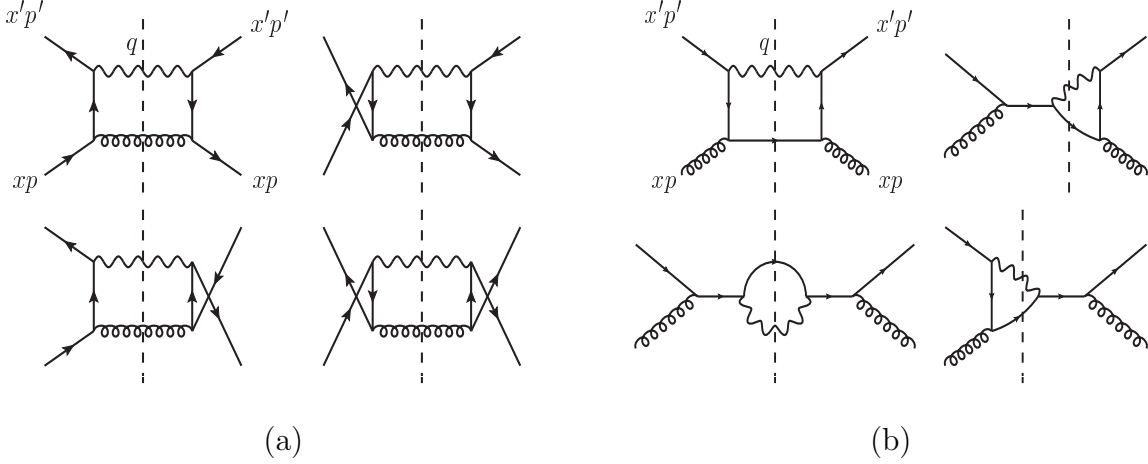


Figure 1: The twist-2 diagrams for the Drell-Yan process. The diagrams in (a) represent the hard part in the $q\bar{q}$ annihilation channel, and those in (b) represent the hard part $\mathcal{H}_{gq, \alpha\beta}^{ab}$ in the quark-gluon scattering channel.

3.2 Twist-3 polarized cross section

The twist-3 cross section for the polarized Drell-Yan process $p^\uparrow(p, S_\perp) + p(p') \rightarrow \gamma^*(q) + X$ is obtained from the initial-state-interaction diagrams shown in Fig. 2. They represent the hard scattering part $S_{\mu\nu\lambda}^{abc}(k_1, k_2, x'p', q)$ defined as the coefficient for the nucleon matrix element proportional to $\langle pS_\perp | A_\nu^b A_\lambda^c A_\mu^a | pS_\perp \rangle$ where k_i ($i = 1, 2$) are the momenta coming out of the polarized nucleon as assigned in Fig. 2. The spin-dependent cross section is obtained by applying the collinear expansion to $S_{\mu\nu\lambda}^{abc}(k_1, k_2, x'p', q)$ with respect to k_i around $x_i p$ where x_i denotes the longitudinal momentum fractions. The diagrams in Fig. 2 give rise to the spin-dependent cross section as a pole contribution at $x_1 = x_2$ from the bared propagator which is referred to as the soft-gluon-pole (SGP) contribution. Applying the formalism developed in [22], one can obtain the polarized cross section induced by the three-gluon correlation functions as

$$\begin{aligned}
\frac{d\Delta\sigma^{\text{DY}}}{dQ^2 dy d^2\vec{q}_\perp} &= \frac{\alpha_{em}^2}{3\pi S Q^2} (-W_\mu^\mu(p, p', q)) \\
&= \frac{\alpha_{em}^2 \alpha_s}{3\pi S Q^2} \sum_q e_q^2 \int \frac{dx'}{x'} f_q(x') \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \left[\frac{\partial S_{\mu\nu\lambda}^{abc}(k_1, k_2, x'p', q) p^\lambda}{\partial k_2^\sigma} \Big|_{k_i=x_i p} \right]^{\text{pole}} \\
&\quad \times \omega_\alpha^\mu \omega_\beta^\nu \omega_\gamma^\sigma M_{F, abc}^{\alpha\beta\gamma}(x_1, x_2),
\end{aligned} \tag{9}$$

where $\omega^\mu_\alpha = g^\mu_\alpha - p^\mu n_\alpha$, and $M_{F,abc}^{\alpha\beta\gamma}(x_1, x_2)$ is the lightcone correlation function of the gluon's field-strengths defined as

$$\begin{aligned} M_{F,abc}^{\alpha\beta\gamma}(x_1, x_2) &= -g(i)^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS | F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) | pS \rangle \\ &= \frac{N d_{bca}}{(N^2 - 4)(N^2 - 1)} O^{\alpha\beta\gamma}(x_1, x_2) - \frac{i f_{bca}}{N(N^2 - 1)} N^{\alpha\beta\gamma}(x_1, x_2) \end{aligned} \quad (10)$$

with $O^{\alpha\beta\gamma}(x_1, x_2)$ and $N^{\alpha\beta\gamma}(x_1, x_2)$ defined in (1) and (2), respectively. For convenience, we have factorized the factor $g\alpha_s$ from $S_{\mu\nu\lambda}^{abc}$ and included g and α_s in the correlation function (10) and the prefactor in (9), respectively. The symbol $[\dots]^{\text{pole}}$ in (9) indicates the pole contribution is to be taken from the hard part. We emphasize that even though the analysis of Fig. 2 starts with the gauge-noninvariant correlation function $\langle pS_\perp | A_\nu^b A_\lambda^c A_\mu^a | pS_\perp \rangle$ and the corresponding hard part $S_{\mu\nu\lambda}^{abc}(k_1, k_2, x'p', q)$, gauge-noninvariant contributions appearing in the collinear expansion either vanish or cancel and the total surviving twist-3 contribution to the single-spin-dependent cross section can be expressed as in (9), using the gauge-invariant correlation functions (1) and (2).

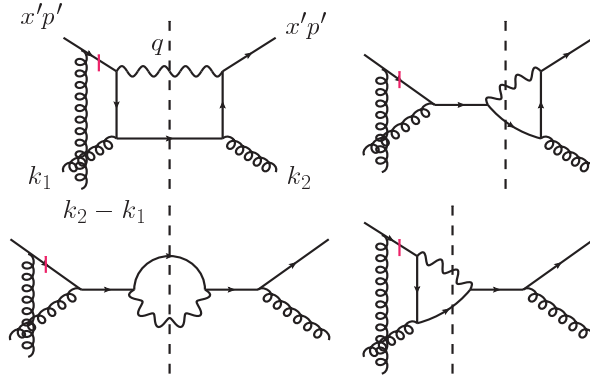


Figure 2: Diagrams representing the three-gluon contribution to the Drell-Yan process. The pole part of the bared propagator in each diagram gives rise to the single-spin-dependent cross section. The mirror diagrams also contribute.

As was shown in [23, 24], one can simplify the actual calculation of the twist-3 cross section by using the “master formula” developed for the three-gluon correlation functions. This simplification occurs due to the fact that the diagrams in Fig. 2 are obtained by attaching the extra gluon-line to those in Fig. 1(b) and the internal quark-propagator next to this attachment gives the pole contribution. For the contribution from the initial-state interaction, the derivative $\left[(\partial/\partial k_2^\sigma) S_{\mu\nu\lambda}^{abc}(k_1, k_2, x'p', q) p^\lambda \right]_{k_i=x_i p}^{\text{pole}}$ is related to the

$gq \rightarrow \gamma^* q$ hard scattering part shown in Fig. 1 (b) as

$$\begin{aligned}
& \left[\frac{\partial S_{\mu\nu\lambda}^{abc}(k_1, k_2, x'p', q)p^\lambda}{\partial k_2^\sigma} \Big|_{k_i=x_i p} \right]^{\text{pole}} \\
&= \left[\frac{-1}{x_2 - x_1 + i\epsilon} \right]^{\text{pole}} \left(\frac{\partial}{\partial(x'p'^\sigma)} - \frac{p'_\sigma p'^\lambda}{p \cdot p'} \frac{\partial}{\partial(x'p'^\lambda)} \right) \mathcal{H}_{gq,\mu\nu}^{abc}(x_1 p, x'p', q) \\
&= i\pi\delta(x_1 - x_2) \frac{d}{d(x'p'^\sigma)} \mathcal{H}_{gq,\mu\nu}^{abc}(x_1 p, x'p', q), \tag{11}
\end{aligned}$$

where $\mathcal{H}_{gq,\mu\nu}^{abc}$ is obtained from $\mathcal{H}_{gq,\mu\nu}^{ab}$ in (8) by inserting an extra color matrix t^c into the place where the extra gluon-line is attached in Fig. 2. In the second expression of (11), the partial derivative with respect to $x'p'^\sigma$ implies the shorthand notation of

$$\frac{\partial}{\partial(x'p'^\sigma)} f(x'p') = \frac{\partial}{\partial r^\sigma} f(r) \Big|_{r \rightarrow x'p'} \tag{12}$$

In the last expression of (11), the on-shell form of $p'^\mu = (p'^+ = \frac{\vec{p}_\perp^2}{2p'^-}, p'^-, \vec{p}_\perp)$ with $\vec{p}_\perp \neq 0$ should be used in taking the derivative, regarding p'^+ as a dependent variable. Since we are considering the cross section in the frame where p and p' are collinear, we shall take the limit $\vec{p}_\perp \rightarrow 0$ after performing the derivative $d/dx'p'^\sigma$. We remind that $\mathcal{H}_{gq,\mu\nu}^{abc}(xp, x'p', q)$ contains the factor $\delta((xp + x'p' - q)^2)$ associated with the on-shell condition for the final unobserved parton, to which the derivative $d/dx'p'^\sigma$ also hits.

Substituting (10) and (11) into (9), one obtains the cross section as

$$\begin{aligned}
\frac{d\Delta\sigma^{\text{DY}}}{dQ^2 dy d^2\vec{q}_\perp} &= \frac{\alpha_{em}^2 \alpha_s}{3\pi S Q^2} \sum_q e_q^2 \int \frac{dx'}{x'} f_q(x') \int \frac{dx}{x^2} (i\pi) \frac{d}{d(x'p'^\gamma)} \mathcal{H}_{gq,\alpha\beta}^{abc}(xp, x'p', q) \Big|_{\vec{p}'_\perp \rightarrow 0} \\
&\times \left[\frac{N d_{bca}}{(N^2 - 4)(N^2 - 1)} O_\perp^{\alpha\beta\gamma}(x, x) - \frac{i f_{bca}}{N(N^2 - 1)} N_\perp^{\alpha\beta\gamma}(x, x) \right], \tag{13}
\end{aligned}$$

where

$$\begin{aligned}
O_\perp^{\alpha\beta\gamma}(x, x) &= 2iM_N \left[O(x, x) g_\perp^{\alpha\beta} \epsilon^{\gamma pn S_\perp} + O(x, 0) (g_\perp^{\beta\gamma} \epsilon^{\alpha pn S_\perp} + g_\perp^{\gamma\alpha} \epsilon^{\beta pn S_\perp}) \right], \\
N_\perp^{\alpha\beta\gamma}(x, x) &= 2iM_N \left[N(x, x) g_\perp^{\alpha\beta} \epsilon^{\gamma pn S_\perp} - N(x, 0) (g_\perp^{\beta\gamma} \epsilon^{\alpha pn S_\perp} + g_\perp^{\gamma\alpha} \epsilon^{\beta pn S_\perp}) \right]. \tag{14}
\end{aligned}$$

From this form, one sees that the twist-3 cross section is written in terms of the four functions $O(x, x)$, $O(x, 0)$, $N(x, x)$ and $N(x, 0)$ as in $ep^\uparrow \rightarrow eDX$ and $p^\uparrow p \rightarrow DX$.

The hard cross sections for $O(x, x)$ and $N(x, x)$ are obtained from the contraction of $\mathcal{H}_{gq,\alpha\beta}^{abc}$ with $g_\perp^{\alpha\beta}$ as in the case of the unpolarized cross section in (7). Taking the color

contraction of $\mathcal{H}_{gq,\alpha\beta}^{abc}$, one obtains ³

$$\begin{aligned} \frac{Nd_{bca}}{(N^2-1)(N^2-4)}\mathcal{H}_{gq,\alpha\beta}^{abc}(xp, x'p', q)g_{\perp}^{\alpha\beta} &= \frac{if_{bca}}{N(N^2-1)}\mathcal{H}_{gq,\alpha\beta}^{abc}(xp, x'p', q)g_{\perp}^{\alpha\beta} \\ &= -\hat{\sigma}_{gq}(\hat{s}, \hat{t}, \hat{u}, Q^2)\delta(\hat{s} + \hat{t} + \hat{u} - Q^2), \end{aligned} \quad (15)$$

where $\hat{\sigma}_{gq}$ is the twist-2 unpolarized cross section in the gq -channel defined in (7). For the scalar functions $\hat{\sigma}_{gq}(\hat{s}, \hat{t}, \hat{u}, Q^2)$ and $\delta(\hat{s} + \hat{t} + \hat{u} - Q^2)$, one can perform the derivative with respect to $x'p'^{\gamma}$ in (13) through that with respect to \hat{u} as

$$\begin{aligned} \frac{d}{d(x'p'^{\gamma})}\hat{\sigma}_{gq}(\hat{s}, \hat{t}, \hat{u}, Q^2)\delta(\hat{s} + \hat{t} + \hat{u} - Q^2) \Big|_{\vec{p}'_{\perp} \rightarrow 0} \\ = -2q_{\gamma}\frac{\partial}{\partial\hat{u}}\hat{\sigma}_{gq}(\hat{s}, \hat{t}, \hat{u}, Q^2)\delta(\hat{s} + \hat{t} + \hat{u} - Q^2). \end{aligned} \quad (16)$$

From (15) and (16), one sees that the hard cross section for $O(x, x)$ and $N(x, x)$ are common and are determined completely from the twist-2 partonic cross section in the gluon-quark scattering channel.

The hard cross sections for $O(x, 0)$ and $N(x, 0)$ are obtained from the contraction of $(d/dx'p'^{\gamma})\mathcal{H}_{gq,\alpha\beta}^{abc}$ with $g_{\perp}^{\beta\gamma}\epsilon^{\alpha pnS_{\perp}} + g_{\perp}^{\gamma\alpha}\epsilon^{\beta pnS_{\perp}}$. To carry out the derivative, we introduce the two fixed vectors $X^{\mu} = (0, 1, 0, 0)$ and $Y^{\mu} = (0, 0, 1, 0)$, and write

$$g_{\perp}^{\beta\gamma} = -X^{\beta}X^{\gamma} - Y^{\beta}Y^{\gamma}. \quad (17)$$

Then the derivative $d/dx'p'^{\gamma}$ hitting the hard part in (13) for $O(x, 0)$ and $N(x, 0)$ can be written as

$$\begin{aligned} \frac{d}{dx'p'^{\gamma}}\mathcal{H}_{gq,\alpha\beta}^{abc}(xp, x'p', q) \left(g_{\perp}^{\beta\gamma}\epsilon^{\alpha pnS_{\perp}} + g_{\perp}^{\gamma\alpha}\epsilon^{\beta pnS_{\perp}} \right) \\ = -X^{\mu}\frac{d}{dx'p'^{\mu}}\mathcal{H}_{gq,\alpha\beta}^{abc}(xp, x'p', q) \left(X^{\beta}\epsilon^{\alpha pnS_{\perp}} + X^{\alpha}\epsilon^{\beta pnS_{\perp}} \right) \\ -Y^{\mu}\frac{d}{dx'p'^{\mu}}\mathcal{H}_{gq,\alpha\beta}^{abc}(xp, x'p', q) \left(Y^{\beta}\epsilon^{\alpha pnS_{\perp}} + Y^{\alpha}\epsilon^{\beta pnS_{\perp}} \right). \end{aligned} \quad (18)$$

To perform the derivative with respect to $x'p'^{\gamma}$, we note that the color contraction of $\mathcal{H}_{gq,\alpha\beta}^{abc}$ can be decomposed by introducing the scalar functions $J_i(\hat{s}, \hat{t}, \hat{u}, Q^2)$ ($i = 1, \dots, 4$) as

$$\frac{Nd_{bca}}{(N^2-1)(N^2-4)}\mathcal{H}_{gq,\alpha\beta}^{abc}(xp, x'p', q) \left(X^{\beta}\epsilon^{\alpha pnS_{\perp}} + X^{\alpha}\epsilon^{\beta pnS_{\perp}} \right)$$

³Note the sign of each term in (14) and (15).

$$\begin{aligned}
&= \frac{if_{bca}}{N(N^2-1)} \mathcal{H}_{gq,\alpha\beta}^{abc}(xp, x'p', q) \left(X^\beta \epsilon^{\alpha pn S_\perp} + X^\alpha \epsilon^{\beta pn S_\perp} \right) \\
&= \left[J_1(\hat{s}, \hat{t}, \hat{u}, Q^2) (q \cdot X) \epsilon^{qpn S_\perp} + J_2(\hat{s}, \hat{t}, \hat{u}, Q^2) \epsilon^{Xpn S_\perp} + J_3(\hat{s}, \hat{t}, \hat{u}, Q^2) (x'p' \cdot X) \epsilon^{qpn S_\perp} \right. \\
&\quad \left. + J_4(\hat{s}, \hat{t}, \hat{u}, Q^2) (q \cdot X) x' \epsilon^{p'pn S_\perp} \right] \delta(\hat{s} + \hat{t} + \hat{u} - Q^2), \tag{19}
\end{aligned}$$

where we have ignored the term proportional to $(p' \cdot X) \epsilon^{p'pn S_\perp}$, since it vanishes as $p'_\perp \rightarrow 0$ after taking the derivative $d/dx'p'^\gamma$. The contraction with $Y^\beta \epsilon^{\alpha pn S_\perp} + Y^\alpha \epsilon^{\beta pn S_\perp}$ is similarly decomposed, using the same functions J_i ($i = 1, \dots, 4$). From (18) and (19), one can perform the derivative to get the cross section. In using the form (19), we remind that J_2 is dimensionless, while $J_{1,3,4}$ have mass dimension -2 .

Using (15), (16), (18) and (19) in (13), and noting that the polarized hard cross section for the antiquark contribution changes sign for the O contribution, the twist-3 polarized cross section can be expressed as

$$\begin{aligned}
&\frac{d\Delta\sigma^{\text{DY}}}{dQ^2 dy d^2\vec{q}_\perp} \\
&= \frac{\alpha_{em}^2 \alpha_s}{3\pi S Q^2} (2\pi M_N) \epsilon^{qpn S_\perp} \sum_q e_q^2 \int \frac{dx'}{x'} f_q(x') \int \frac{dx}{x^2} \delta(\hat{s} + \hat{t} + \hat{u} - Q^2) \\
&\quad \times \left[\frac{2}{\hat{u}} \delta_q \left\{ Q^2 \left(\frac{\partial \hat{\sigma}_{gq}}{\partial Q^2} + \frac{\partial \hat{\sigma}_{gq}}{\partial \hat{t}} \right) O(x, x) - \hat{\sigma}_{gq} \left(x \frac{dO(x, x)}{dx} - 2O(x, x) \right) \right\} \right. \\
&\quad \left. - \{ \delta_q \rightarrow 1, O(x, x) \rightarrow N(x, x) \} \right. \\
&\quad \left. + \frac{2}{\hat{u}} \delta_q \left\{ \frac{(\hat{t} - Q^2)(\hat{u} - Q^2)}{\hat{s}} - Q^2 \right\} \left\{ \left(J_1 + Q^2 \left(\frac{\partial J_1}{\partial Q^2} + \frac{\partial J_1}{\partial \hat{t}} \right) \right) O(x, 0) \right. \right. \\
&\quad \left. \left. - J_1 \left(x \frac{dO(x, 0)}{dx} - 2O(x, 0) \right) \right\} \right. \\
&\quad \left. + \frac{2}{\hat{u}} \delta_q \left\{ -Q^2 \left(\frac{\partial J_2}{\partial Q^2} + \frac{\partial J_2}{\partial \hat{t}} \right) O(x, 0) + J_2 \left(x \frac{dO(x, 0)}{dx} - 2O(x, 0) \right) \right\} \right. \\
&\quad \left. - \delta_q (2J_3 + J_4) O(x, 0) \right. \\
&\quad \left. + \{ \delta_q \rightarrow 1, O(x, 0) \rightarrow N(x, 0) \} \right], \tag{20}
\end{aligned}$$

where $\delta_q = 1$ for the quark contribution and $\delta_q = -1$ for the antiquark contribution. In writing down (20), we have used the relations $\vec{q}_\perp^2 = (\hat{t} - Q^2)(\hat{u} - Q^2)/\hat{s} - Q^2$, and

$$\hat{s} \frac{\partial J_1}{\partial \hat{s}} + \hat{t} \frac{\partial J_1}{\partial \hat{t}} + \hat{u} \frac{\partial J_1}{\partial \hat{u}} + Q^2 \frac{\partial J_1}{\partial Q^2} + J_1 = 0,$$

$$\hat{s}\frac{\partial J_2}{\partial \hat{s}} + \hat{t}\frac{\partial J_2}{\partial \hat{t}} + \hat{u}\frac{\partial J_2}{\partial \hat{u}} + Q^2\frac{\partial J_2}{\partial Q^2} = 0, \quad (21)$$

which follows from the scale invariant properties $J_2(\lambda\hat{s}, \lambda\hat{t}, \lambda\hat{u}, \lambda Q^2) = J_2(\hat{s}, \hat{t}, \hat{u}, Q^2)$ and a similar relation for $Q^2 J_1(\hat{s}, \hat{t}, \hat{u}, Q^2)$. By the explicit calculation of $J_i(\hat{s}, \hat{t}, \hat{u}, Q^2)$ ($i = 1, \dots, 4$), one obtains the twist-3 polarized cross section induced by the three-gluon correlation functions as

$$\begin{aligned} & \frac{d\Delta\sigma^{\text{DY}}}{dQ^2 dy d^2\vec{q}_\perp} \\ &= \frac{2M_N\alpha_{em}^2\alpha_s}{3SQ^2} \int \frac{dx}{x} \int \frac{dx'}{x'} \delta(\hat{s} + \hat{t} + \hat{u} - Q^2) \epsilon^{qpnS_\perp} \frac{1}{\hat{u}} \sum_q e_q^2 f_q(x') \\ &\times \left[\delta_q \left\{ \left(\frac{dO(x, x)}{dx} - \frac{2O(x, x)}{x} \right) \hat{\sigma}_1 + \left(\frac{dO(x, 0)}{dx} - \frac{2O(x, 0)}{x} \right) \hat{\sigma}_2 + \frac{O(x, x)}{x} \hat{\sigma}_3 + \frac{O(x, 0)}{x} \hat{\sigma}_4 \right\} \right. \\ &\left. - \left(\frac{dN(x, x)}{dx} - \frac{2N(x, x)}{x} \right) \hat{\sigma}_1 + \left(\frac{dN(x, 0)}{dx} - \frac{2N(x, 0)}{x} \right) \hat{\sigma}_2 - \frac{N(x, x)}{x} \hat{\sigma}_3 + \frac{N(x, 0)}{x} \hat{\sigma}_4 \right], \end{aligned} \quad (22)$$

where the partonic hard cross sections are given by

$$\left\{ \begin{aligned} \hat{\sigma}_1 &= \frac{2}{N} \left(\frac{\hat{u}}{\hat{s}} + \frac{\hat{s}}{\hat{u}} + \frac{2Q^2\hat{t}}{\hat{s}\hat{u}} \right), \\ \hat{\sigma}_2 &= \frac{2}{N} \left(\frac{\hat{u}}{\hat{s}} + \frac{\hat{s}}{\hat{u}} + \frac{4Q^2\hat{t}}{\hat{s}\hat{u}} \right), \\ \hat{\sigma}_3 &= -\frac{1}{N} \frac{4Q^2(Q^2 + \hat{t})}{\hat{s}\hat{u}}, \\ \hat{\sigma}_4 &= -\frac{1}{N} \frac{4Q^2(3Q^2 + \hat{t})}{\hat{s}\hat{u}}. \end{aligned} \right. \quad (23)$$

For a large Q^2 , $\hat{\sigma}_1$ differs from $\hat{\sigma}_2$ significantly, and $\hat{\sigma}_{3,4}$ are not negligible. Therefore the cross section depends on the four functions $O(x, x)$, $O(x, 0)$, $N(x, x)$ and $N(x, 0)$ independently as in the case of $ep^\uparrow \rightarrow eDX$ [22].

Our derivation of (22) and (23) is based on the master formula (11). Alternatively, one can directly calculate the derivative $\left[(\partial/\partial k_2^\sigma) S_{\mu\nu\lambda}^{abc}(k_1, k_2, x'p', q) p^\lambda \right]_{k_i=x_{ip}}^{\text{pole}}$ in (9) to get the cross section. We have performed such calculation and confirmed that the result agrees with (22).

4 Direct photon production

4.1 Spin-dependent cross section

The cross section formula for the direct photon production can be easily obtained by taking the $Q^2 \rightarrow 0$ limit of the result for the Drell-Yan process. The twist-2 unpolarized cross section for $p(p) + p(p') \rightarrow \gamma(q) + X$ is given by

$$E_\gamma \frac{d\sigma^\gamma}{d^3\vec{q}} = \frac{\alpha_{em}\alpha_s}{S} \sum_q e_q^2 \int \frac{dx'}{x'} \int \frac{dx}{x} \left[f_q(x) f_{\bar{q}}(x') \hat{\sigma}_{q\bar{q}}^\gamma + G(x) f_q(x') \hat{\sigma}_{gq}^\gamma \right. \\ \left. + f_q(x) G(x') \hat{\sigma}_{qg}^\gamma \right] \delta(\hat{s} + \hat{t} + \hat{u}), \quad (24)$$

where E_γ is the energy of the photon and the partonic hard cross sections are defined as

$$\hat{\sigma}_{q\bar{q}}^\gamma = \frac{2C_F}{N} \left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} \right), \quad \hat{\sigma}_{gq}^\gamma = -\frac{1}{N} \left(\frac{\hat{u}}{\hat{s}} + \frac{\hat{s}}{\hat{u}} \right), \quad \hat{\sigma}_{qg}^\gamma = -\frac{1}{N} \left(\frac{\hat{t}}{\hat{s}} + \frac{\hat{s}}{\hat{t}} \right), \quad (25)$$

with \hat{s} , \hat{t} and \hat{u} in (6) with $q^2 = 0$. Likewise the twist-3 polarized cross section for $p^\uparrow(p, S_\perp) + p(p') \rightarrow \gamma(q) + X$ is given by

$$E_\gamma \frac{d^3\Delta\sigma^\gamma}{d^3\vec{q}} = \frac{4\alpha_{em}\alpha_s\pi M_N}{S} \sum_q e_q^2 \int \frac{dx'}{x'} f_q(x') \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u}) \epsilon^{qpmS_\perp} \left(\frac{-1}{\hat{u}} \right) \\ \times \left[\delta_q \left(\frac{dO(x)}{dx} - \frac{2O(x)}{x} \right) - \left(\frac{dN(x)}{dx} - \frac{2N(x)}{x} \right) \right] \hat{\sigma}_{gq}^\gamma, \quad (26)$$

where

$$O(x) = O(x, x) + O(x, 0), \quad N(x) = N(x, x) - N(x, 0). \quad (27)$$

This result differs from the previous one in [19], in which the calculation was not based on the factorization formula (9). Compared with the Drell-Yan case (22), this cross section has a much simpler form; it depends on the three-gluon correlation functions only through the combinations of $dO(x)/dx - 2O(x)/x$ and $dN(x)/dx - 2N(x)/x$, accompanying the common partonic hard cross section which is the same as the twist-2 hard cross section for the $gq \rightarrow \gamma q$ scattering in (25). This is in parallel with the twist-3 cross section for $p^\uparrow p \rightarrow DX$ in the $m_c \rightarrow 0$ limit. (At the RHIC energy, the effect of $m_c \neq 0$ is negligible and thus one could regard the cross section as a function of $O(x)$ and $N(x)$ [24].) We also remind that the SGP contribution from the quark-gluon correlation function $G_F(x, x)$ to $p^\uparrow p \rightarrow \pi X$ also appears in the combination $dG_F(x, x)/dx - G_F(x, x)/x$, whose origin was clearly understood in terms of the master formula [13].

From (26), one sees that if $dO(x)/dx - 2O(x)/x$ and $dN(x)/dx - 2N(x)/x$ have the same sign and a similar magnitude, the quark contribution from the unpolarized nucleon cancel between the two contributions, and only the antiquark-contribution is active, which would

lead to a small asymmetry. On the other hand, if $dO(x)/dx - 2O(x)/x$ and $dN(x)/dx - 2N(x)/x$ have the opposite sign, the quark-contribution become active and thus one would expect a large asymmetry. Therefore, A_N^γ can be an important measure for the relative sign and magnitude of the three-gluon correlation functions.

4.2 Numerical calculation of the asymmetry

In the previous section, we saw that the spin-dependent cross section for the direct-photon production depends on the two combinations $O(x)$ and $N(x)$ in (27). These are the same functions which govern the SSA for the $p^\uparrow p \rightarrow DX$ [24]. Here we illustrate the impact of the three-gluon correlation function on the asymmetry $A_N^\gamma \equiv \Delta\sigma^\gamma/\sigma^\gamma$ based on the same models used in the study of $p^\uparrow p \rightarrow DX$.

In [24], we have calculated A_N^D at the RHIC energy with the two models for $O(x)$ and $N(x)$: (Model 1); $O(x) = K_G x G(x)$ and (Model 2); $O(x) = K'_G \sqrt{x} G(x)$ with the gluon density in the nucleon $G(x)$ assuming $O(x) = N(x)$. The constants K_G and K'_G were determined to be $K_G = 0.004$ and $K'_G = 0.001$ so that the calculated A_N^D are consistent with the preliminary data of RHIC.⁴ For the case of D -meson, change of the relative sign between $O(x)$ and $N(x)$ causes the interchange of A_N^D between D and \bar{D} mesons. To calculate A_N^γ , we also use the above two models for the two cases: (Case 1); $O(x) = N(x)$ and (Case 2); $O(x) = -N(x)$. We use the GJR08 [31] for the unpolarized distributions in the nucleon and calculate A_N^γ as a function of $x_F = 2q_\parallel/\sqrt{S}$ at the RHIC energy of $\sqrt{S} = 200$ GeV and the transverse momentum of the photon $q_T = 2$ GeV, setting the scale of all the distribution functions at $\mu = q_T$.

Fig. 3 shows the result for A_N^γ for each case. One can see A_N^γ at $x_F > 0$ become almost zero regardless of the magnitude of the three-gluon correlation functions, while A_N^γ at $x_F < 0$ depends strongly on the relative sign between $O(x)$ and $N(x)$ and also the small- x behavior of the three-gluon correlation functions as in the case of $p^\uparrow p \rightarrow DX$. Even though the derivatives of $O(x)$ and $N(x)$ contribute, A_N^γ is tiny at $x_F > 0$ due to the small partonic cross section which occurs from the u -channel diagrams. At $x_F < 0$, large- x' region of the unpolarized quark distributions and the small- x region of the three-gluon correlation functions are relevant. For the above case 1, only antiquarks in the unpolarized nucleon are active and thus leads to small A_N^γ as shown in Figs. 3(a) and (b). On the other hand, for the case 2, quarks in the unpolarized nucleon are active and thus lead to large A_N^γ as shown in Figs. 3(c) and (d). Therefore A_N^γ at $x_F < 0$ for the direct photon production could provides us with an important information on the relative sign between $O(x)$ and $N(x)$.

In the present calculation of A_N^γ , we have included only the three-gluon correlation functions for the spin-dependent cross section. For the complete calculation, one has to include the contribution from the quark-gluon correlation functions. The quark-gluon correlation functions, however, gives rise to A_N^γ only in the positive x_F region [32] as in the case of

⁴In [24], we employed an ansatz, $O(x, x) = K_G x G(x)$ and $O(x, x) = K'_G \sqrt{x} G(x)$ with the relation $O(x, x) = O(x, 0) = N(x, x) = -N(x, 0)$ and thus K_G and K'_G in this paper are twice as large as those in [24].

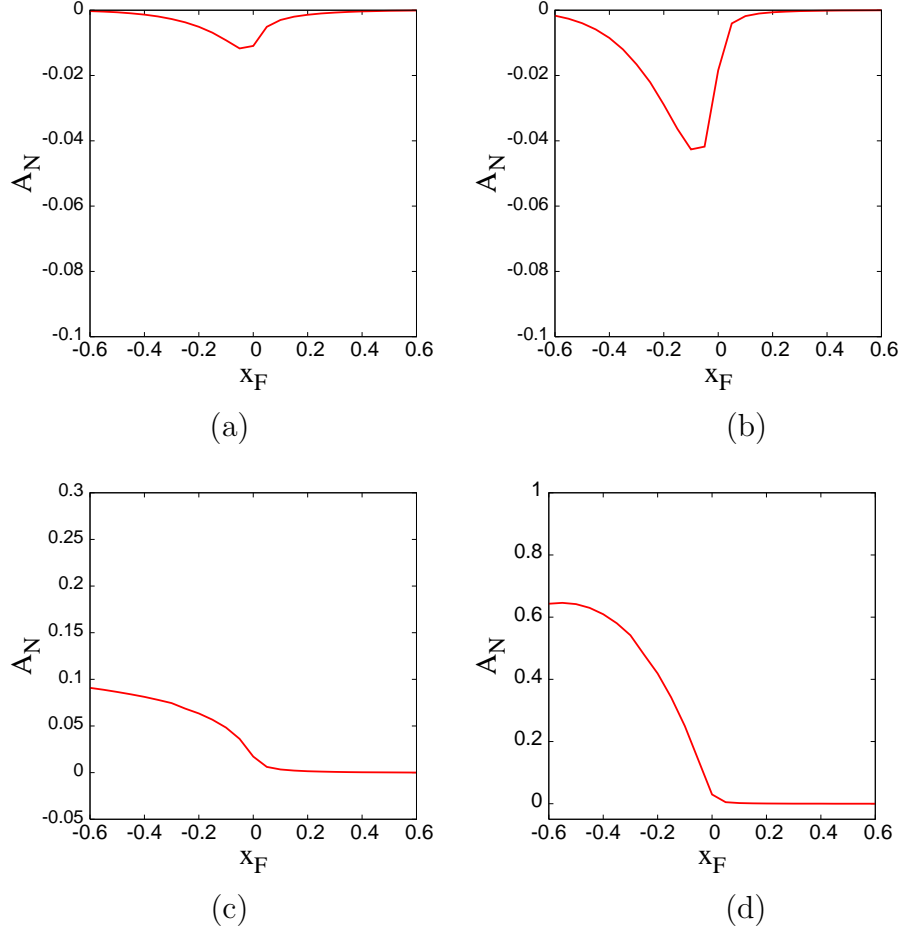


Figure 3: A_N^γ as a function of x_F at the RHIC energy $\sqrt{S} = 200$ GeV and $q_T = 2$ GeV. (a) A_N^γ for the case 1 with the model 1. (b) A_N^γ for the case 1 with the model 2. (c) A_N^γ for the case 2 with the model 1. (d) A_N^γ for the case 2 with the model 2.

A_N for $p^\uparrow p \rightarrow \pi X$. Therefore, in A_N^γ , the role of three-gluon correlation functions and the quark-gluon correlation functions are clearly separated and A_N^γ at $x_F < 0$ truly plays an important role to probe the three-gluon correlation functions.

5 Summary

In this paper, we have derived the twist-3 single-spin-dependent cross section for the Drell-Yan lepton-pair production and the direct-photon production in the pp collision induced by the three-gluon correlation functions in the LO QCD. The derivation was carried out by using the master formula for the three-gluon correlation functions, which simplifies the calculation and makes connection between the corresponding hard cross section and the $gq \rightarrow \gamma^{(*)}q$ hard scattering in the twist-2 level. The cross section for the Drell-Yan process

receives the contribution from the four functions $O(x, x)$, $O(x, 0)$, $N(x, x)$ and $N(x, 0)$ with different hard cross sections as in the case of the SIDIS, $ep^\uparrow \rightarrow eDX$. For the direct-photon production, the cross section is expressed in terms of the particular combinations $O(x, x) + O(x, 0)$ and $N(x, x) - N(x, 0)$ with the common hard cross section which is the same as the twist-2 unpolarized hard cross section in the $gq \rightarrow \gamma q$ scattering channel. This simplification is the same as those for the three-gluon contribution to $p^\uparrow p \rightarrow DX$ in the $m_c \rightarrow 0$ limit [24] and the SGP contribution of the quark-gluon correlation function to $p^\uparrow p \rightarrow \pi X$ [13]. To illustrate the impact of three-gluon correlation functions, we have performed a model calculation for SSA in the direct-photon production A_N^γ , using the model determined from the SSA data for $p^\uparrow p \rightarrow DX$ at RHIC. It turned out that the effect of the three-gluon correlation functions is negligible in the forward direction of the polarized nucleon, but can be substantial in the backward direction, depending on the small- x behavior of the three-gluon correlation functions and the relative sign of the two functions $O(x, x) + O(x, 0)$ and $N(x, x) - N(x, 0)$. Since the effect of the quark-gluon correlation on SSA is negligible in the backward direction, the SSA in that region provides us with an important information on the three-gluon correlation.

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